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Spontaneous focusing on numerosity and mathematical skills of young children

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Abstract

Two studies were conducted to investigate, firstly, children's focusing on the aspect of numerosity in utilizing enumeration in action, and, secondly, whether children's Spontaneous FOcusing on Numerosity (SFON) is related to their counting development. The longitudinal data of 39 children from the age of 3.5 to 6 years showed individual differences in SFON, as well as stability in children's SFON across tasks during the follow-up. Path analyses indicated a reciprocal relationship between SFON and counting development. The results were confirmed by a cross-sectional study of 183 6.5-year-old children when the effects of non-verbal IQ, verbal comprehension and lacking enumeration and procedural skills were controlled.

1. Introduction

The basic idea of our study is that in order to utilize knowledge about numerosity in action, attention has to be focused on numerosity as a property of a set. We argue that this intentional separate sub-process in enumeration is one of the essential factors developing in interaction with early mathematical skills during early childhood. For a young child it is not self-evident to regard numerosity as a relevant

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factor in tasks. Although children's mathematical development, especially their counting development has been widely studied, focusing on the aspect of numerosity has earlier been neither clearly conceptualized nor sufficiently studied in relation to early mathematical skills.

In principle, infants, as well as many animals, have some biologically primary quantitative abilities, which encompass their implicit understanding of numerosity, ordinality, counting, and simple arithmetic, and thus form the basis for later mathematical development (for reviews, see Geary, 2000; Wynn, 1998). However, according to many studies, conceptual and procedural proficiency in counting takes a considerably long time to develop from the first number words and pointing trials to mastery of accurate counting and a full understanding of the goals of quantifying in determining the cardinality of a set in a variety of tasks (e.g. Fuson, 1988; Sophian & Kailihiwa, 1998; Wynn, 1990). This might indicate not only well documented differences in quantification systems for small and large sets (e.g. Dehaene & Cohen, 1994; Kaufman, Lord, Reese, & Volkman, 1949; Sathian et al., 1999), but also the need for versatile and long-term practice in counting. Previous studies (e.g. Starkey & Cooper, 1980) have demonstrated that all children have pre-attentional mechanisms for recognizing small numerosity. However, the results of Hannula and Lehtinen (2001, 2003) showed that a substantial group of young children did not spontaneously focus on the aspect of numerosity and utilize knowledge concerning small numbers of objects, although they had the enumeration skills required for the tasks. Further theoretical as well as methodological clarity, first, in distinguishing, focusing on numerosity as a separate mental process from exact enumeration, and second, in investigating individual differences in children's spontaneous (i.e. non-guided) focusing on numerosity in relation to their counting development, could provide us with a greater understanding of children's mathematical development.

In natural situations, exact number recognition demands carving up the set of objects on some reasonable basis and focusing on the aspect of numerosity in the set of objects. A person needs to select the set of objects, or which kind of objects especially, she or he is interested in (Wynn, 1998). Not all the possible numerosity in a scene can be brought to the conscious level of processing. The numerosity of items depends on the way one carves up the set of items and, thus, on the goal of quantification.

Recent findings of Ganel and Goodale (2003) show that even for utilizing knowledge about the shape of objects in activities requires analytical, conscious processes whereas perception of the same aspects happens in a holistic way. Thus, here we propose that utilizing the aspect of exact numerosity in action does not occur automatically, but requires intentional processes. This is in line with Trick and Pylyshyn's (1994) theory on subitizing (i.e. the rapid apprehension of small numerosity). According to this theory, enumeration by subitizing *makes use* of the pre-attentional process, producing only pre-numeric individuation information about objects in the set, but requires also goal-directed processes. Subitizing-based enumeration occurs when one *decides* to enumerate, and the number of items is small enough. In order to recognize the cardinal values of objects and to take into account significant numerosity in the action, the child needs to intentionally focus his or her attention on the numerical aspect of the target objects; not only to carve up the set of target objects, but also to activate the numerical recognition stage, in which the choice of numeric response is made. Subitizing-based enumeration has been traditionally measured with tasks in which the subject's attention has been guided towards the aspect of numerosity (for review, see Trick & Pylyshyn, 1994), so these studies do not show subitizing-based enumeration happening automatically. It is more commonly accepted that counting objects or incidents one by one requires intentional activation of the counting processes.

The present study addresses the role of children's spontaneous practice in utilizing exact enumeration in counting development. It is not only the socio-cultural mediation of numerical cognition which develops the child's skill to focus on the aspect of numerosity and to utilize innate and cultural tools for enumeration, but also focusing on the aspect of numerosity develops the child's enumeration skills by activating the enumeration process and thus producing practice in enumeration.

Ericsson and Lehman (1996) show that experts seem to be capable of "seeing" multiple possibilities to practice their skills in everyday situations, and this has been an essential part of their development from a very early age. In the same way, we assume that children's own spontaneous focusing on numerosity frequently leads them to perceive different numbers of objects or events in their surroundings, and thus they get practice in recognizing and producing numerosity. This, in turn, develops their quantifying skills in several ways: not only broadening their counting range, but also, along with more developed counting (quantifying) skills, a larger quantity may appear as a possible subject for counting. Moreover, knowledge about the use of quantifying means in different tasks will increase with practice, so the child may tend to focus on numerosity more in new, more demanding tasks. As Lindahl and Samuelsson (2002) state, when children's intention is focused on specific phenomena, they look for a variety of situations where they can practice the phenomenon they want to master.

The same items in the child's environment arouse different interpretations: props for naming, mouthing and banging as well as quantification (Gelman, 1990). Which action occurs in a given setting is determined mostly by the child's goals, but it is also affected by the child's skills and concepts. Sophian (1998) has suggested that children's conceptual knowledge about numbers is dynamically related to their goal-based numerical activities: conceptual advances facilitate new goals and corresponding activities, which in turn provide the input for further advances. According to the reciprocal views of Saxe, Guberman, and Gearhart (1987) and Sophian (1998) on counting development, socially structured goals of quantification change along with the development of skills and direct children's attention to different aspects of numbers and the ways in which others use them.

The aim of our research is to study young children's Spontaneous FOcusing on Numerosity (SFON). We assume that within the frames of a child's existing mathematical competence it is possible to distinguish a separate mental process, one which refers to the child's tendency to spontaneously focus on the aspect of numerosity and utilize his or her enumeration skills in various activity situations. We hypothesize that there should be some stability in child's spontaneous focusing on numerosity across different tasks, and across time. Furthermore, SFON tendency should be related to the development of mathematical skills during the phase when children normally establish their counting skills. The association between SFON and mathematical skills should not be explained by children's differences in non-verbal IQ, comprehension of verbal instructions nor by their lack of enumeration or other cognitive skills required for executing the SFON tasks.

2. Study 1

The aims of Study 1 were to examine children's SFON longitudinally and to study how it was related to the formation of early mathematical skills. Our hypothesis was that young children vary in their SFON and that there is some stability in this tendency. In addition, we assume that SFON is involved in the development of mathematical skills. A strong SFON tendency might result in an improvement in mathematical skills, and well developed mathematical skills could in turn increase children's subsequent spontaneous tendency to focus on numerosity.

2.1. Method

2.1.1. Participants

Participants were 39 children with no developmental delays (18 girls and 21 boys) representing urban children from Finnish-speaking families in day care. The children's ages ranged from 3.3 to 3.7 years ($M = 3.6$ years, SD = 1.5 months) when the data collection began. The personnel of the day-care centres and the parents were informed that the focus of the study would be on children's general quantitative development. No specifications were made. Data collection was complete, resulting in no missing data.

2.1.2. Design

The children's SFON was tested with an exact interval of one year, at the ages of 4, 5 and 6, for their mathematical skills, i.e. for number sequence production skill at 3.5, 5 and 6, for their early cardinality skills at 3.5, and for their object counting skills at 5 and 6. The first author presented all the tasks except for the SFON tasks at 6. The tasks were presented in the same order for all children.

2.1.3. Tasks

2.1.3.1. SFON. All the testing sessions were carried out in a familiar room in the child's day-care centre. When presenting the SFON tasks, no use was made of any phrase which could have suggested that the tasks were somehow mathematical or quantitative. The tasks included only very small numerosities, which all children should be able to handle.

All the child's (a) utterances including number words (e.g. "I'll give him two berries"), (b) use of fingers to express numbers, (c) counting acts, like a whispered number word sequence and indicating acts by fingers and/or head, (d) other comments referring either to quantities or counting (e.g. "Oh, I miscounted them"), or, (e) interpretation of the goal of the task as quantitative (e.g. "I gave exactly accurate number of them") were identified. The child was scored as focusing on numbers, if she or he produced the correct numerosity, and/or, if she or he was observed presenting any of the aforementioned (a–e) quantifying acts. The scoring was based on analyses of video-recorded task situations. The SFON tasks were designed to become progressively more difficult across the measurement points as children became older. The maximum score on every SFON task was 3.

2.1.3.1.1. Imitation tasks at the ages of 4 and 5 years. The materials of the Imitation task at 4 were a toy parrot, capable of swallowing, placed in front of the child on the table, and a plate of red glass berries (2 cm in diameter) placed in front of the parrot. At 5 the materials were a bear-shaped savings-box with a hole for money in the back, and a box of Finnish Marks. The testing procedure at 5 corresponded to that used at 4 except for the introduction of the material.

The experimenter started the task by introducing the materials and then said: "Watch carefully what I do, and then you do just like I did." The experimenter put two berries one at a time into the parrot's mouth, and they disappeared with a bumping sound into the parrot's stomach. Then the child was told: "Now you do exactly like I did". The number of berries in the second item depended on the success of the first item. For those who gave the parrot two berries in the first item, the second item was carried out with three berries. The others were again confronted with two berries as a second item. For the third item, one berry was used for everyone. In the 5-year-olds' task, the number of coins across the items was 2, 2, and 1, respectively.

2.1.3.1.2. Imitation task at the age of 6 years. The materials of the Imitation task were a post-box, placed in front of the child on the table, and a pile of 10 red closed blank envelopes placed on the left, in front of the box. A pile of 10 blue envelopes was placed on the right of the red envelopes 10 cm from the red pile.

The experimenter started the task by pointing to the piles of envelopes, and saying: "This is a post-box, and there are red envelopes here and blue envelopes here. Watch carefully what I do, and then you do just like I did." The experimenter put 2 red envelopes one at a time and 1 blue envelope into the post-box. Then the child was told: "Now you do exactly like I did". For the second test item, 2 red and 3 blue envelopes, and for the third item, 3 blue and 2 red envelopes were used.

2.1.3.1.3. Model task at the age of 6 years. The materials of the Model task were A4-sized pictures of dinosaurs, and three stamps: a circular stamp called "node" and two different triangular stamps called "spikes". Similar pictures of dinosaurs were placed in front of the experimenter and the child. The child was told that in this task the experimenter would make her dinosaur into a model (see Fig. 1) and then she turns the model upside down. Then the child has to make his or her dinosaur look exactly like the model one. After introducing the task in this way, the experimenter said: "Now, watch carefully, I am making this dinosaur into a model." After

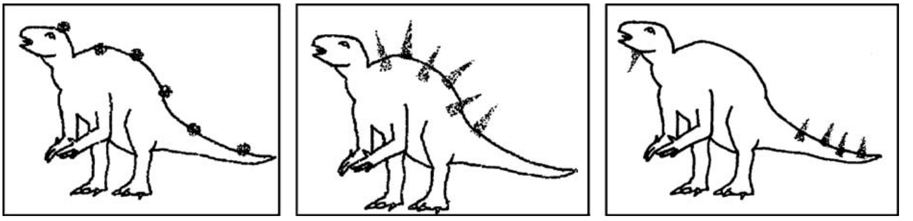

Fig. 1. The models of the 1st, 2nd, and 3rd item in the Model task.

stamping 6 nodes starting from the head of the dinosaur, the model was turned upside down by the experimenter, who then gave the stamp to the child, and said: "Now, make your dinosaur look like the model dinosaur." The procedure was repeated with the 7-spiked dinosaur as the second item and the 5-spiked dinosaur as the third item.

2.1.3.1.4. Finding task at the age of 6 years. This task involved 27 tapering wooden hats (diameter 16 mm at the bottom, and height 16 mm) side by side in a semicircle (28 cm in diameter) on a mat, and a small troll and his gold ingot (4 mm × 4 mm × 1 mm).

The experimenter introduced the materials and said: "Now you may watch while I hide this gold ingot under a hat. Then you can tell the troll where his gold ingot is hidden. Now watch under which hat I hide the gold ingot." The experimenter lifted the target hat into her hand and placed the gold ingot in the place of the hat. Then the experimenter counted silently to four giving the child time to imprint on his/her mind the location of the gold ingot, after which the experimenter covered the gold ingot with the hat. To prevent the child's staring at the target hat, the child was asked to look at the right and left corners of the ceiling. The child was not allowed to use his/her finger to mark the target hat. Then the child was asked to show under which hat the gold ingot was. If the child failed to find the gold ingot, the experimenter lifted the target hat, and removed the gold ingot in order to start the next trial. The ingot was hidden firstly under the 6th hat from the right, secondly under the 7th hat from the left, and thirdly under the 5th hat from the right.

After all three trials, the child was interviewed about how she or he had tried to remember where the gold ingot was hidden. It was recorded from the interview whether the child had used counting or otherwise had known either the number of hats before the target hat, or the ordinal number of the target hat. The scoring of the task was based on analyses of video-recordings, the interview, and the ordinal number of the hat pointed at by the child. If the analyses of the child's behaviour and the interview revealed that the child had found the target hat by accident, not on the basis of any knowledge of the exact number of hats, no scores of focusing on numerosity were given, irrespective of whether the target hat was found or not.

For the further analyses, the average score of the three SFON tasks presented at age 6 was used as an indicator of the children's SFON tendency at 6. The mean of the 6-year-olds' average SFON score was 1.25 (SD = 0.68). The intraclass correlation for the three 6-year-olds' tasks was $r = 0.43$, indicating some differences in task demands.

2.1.3.2. Mathematical skills

2.1.3.2.1. Cardinality skills. Early cardinality recognition and production skills were measured by a caterpillar task (for a more detailed description, see Hannula & Lehtinen, 2001). The materials in the caterpillar task were 10 cloth "caterpillars" (length 60 cm), with 1, 2, 3..., or 10 legs (length 6 cm) at 2.5 cm intervals, and a small box of socks for the caterpillars.

The child was introduced to a frame story about caterpillars needing socks before going out, and a model caterpillar with socks already on. Then the child was asked to bring the next 2-legged caterpillar as many socks as needed from the box on the opposite side of the table. All the socks the caterpillar needed were to be brought at the same time. It was carefully confirmed that the child understood the quantitative goal of the task, i.e. bringing exactly the correct number of socks to the caterpillar. The caterpillars were presented in the order 2, 1, 3, 4, 2, 5, 6, 1, 7, 8, 2, 9 and 10, where the numbers represent the number of legs of the caterpillars. If the child did not bring the right number of socks for the caterpillar, the failed number was repeated, and if the child failed twice with the same number, the previous successful number was repeated. The number, at which the child twice brought the correct number of socks to the caterpillar, determined the upper range of the child's cardinality skills. The maximum score on the task was 10.

2.1.3.2.2. Number word sequence production. The skill to produce a number word sequence was determined by the highest number up to which the child could accurately count from one on, aloud, in one of the two trials. The child's counting was stopped at 50, the maximum score on the task.

2.1.3.2.3. Counting of disarranged objects. The child was asked to carefully count aloud how many objects there were on the table. The randomly set out movable objects placed were wooden, painted pictures (each about 3.4 cm in diameter). The sets of objects of the same kind were presented in the order 3, 5, 7, 9, 13, 19, 23. The highest number at which the child counted accurately determined the level of the child's performance. The counting was interpreted as accurate if the child said all the necessary number words in the right order, pointed once to every countable object, and did not violate the one to one correspondence between number words and pointing. Counting without pointing was considered accurate if the child's number word sequence and cardinal number word were accurate. The maximum score on the task was 23.

2.1.3.2.4. Mathematical sum variables. We formed three sum variables for mathematical skills by averaging the z-scores of number sequence production and cardinality skills at 3.5, number sequence production and object counting at 5, as well as number sequence production and object counting at 6. The correlations of the mathematical measures were at 3.5 $r = 0.69$, at 5 $r = 0.65$, and at 6 $r = 0.51$. The distributions of the created sum variables were satisfactory (see Table 2).

2.2. Results

Table 1 presents the numbers of children focusing on numerosity from zero to three times in individual SFON tasks. Together with the descriptive statistics of the observed variables in Table 2, the data show that there were remarkable individual differences in children's spontaneous focusing on numerosity at every measurement point. The variation in difficulty of the individual SFON tasks is also noteworthy.

In order to study the stability of children's SFON during the follow-up, the average intraclass correlation for SFON at 4, 5 and 6, $r = 0.59$, was calculated. Thus, it seems that in spite of different task contexts, the 2-year time period, and measures within subitizable and counting ranges, there was reasonable stability in children's spontaneous focusing on numerosity.

The children's mathematical skills developed significantly during the follow-up (Table 2) from an average number sequence production skill of counting to 9.51 and recognizing the cardinality of 3.00 objects to the average number sequence production skill of counting to 36.38 and counting 14.18 objects.

Correlations between the sum variables in Table 3 show that SFON at 4, 5 and 6 was positively associated with earlier, concurrent and subsequent mathematical skills at all measurement points, except for a non-significant association between SFON at 5 and mathematical skills at 6. The correlations also show clear consistency in mathematical skills during the follow-up, as well as in SFON from 4 to 5 years. Pearson correlation of SFON at 6 did not show significant correlation with earlier SFON measures, although there was a significant intraclass correlation between all the SFON measures.

Next, path analysis was used to analyse the relationships between SFON and mathematical skills from 3.5 to 6 years. This was done with LISREL 8.51 (Jöreskog & Sörbom, 2001), using the maximum likelihood method in fitting the models to the covariance matrix. In evaluating the goodness of fit of the models to the sample data, we used not only the traditional Chi-Square, but also two indicators recommended for small samples by Hu and Bentler (1999): the Standardized Root Mean Square Residual (SRMR) and the Comparative Fit Index (CFI). Hu and Bentler (1999)

Table 1
Frequencies of spontaneous focusing on numerosity in the SFON tasks of Study 1 ($N = 39$)

Number of SFON scores	Imitation at 4 years: bird	Imitation at 5 years: savings-box	Imitation at 6 years: post-box	Model task at 6 years	Finding task at 6 years
0	10 (26%)	17 (44%)	7 (18%)	11 (28%)	17 (44%)
1	7 (18%)	4 (10%)	6 (15%)	21 (54%)	12 (31%)
2	6 (15%)	2 (5%)	12 (31%)	6 (15%)	4 (10%)
3	16 (41%)	16 (41%)	14 (36%)	1 (3%)	6 (15%)

Table 2
Descriptives of the variables in Study 1

Variable	*M*	SD	Skewness	Kurtosis	Range
SFON tasks					
Imitation					
Bird at 4 years	1.72	1.26	−0.27	−1.61	0–3
Savings-box at 5 years	1.44	1.41	0.11	−1.94	0–3
Post-box at 6 years	1.85	1.11	−0.53	−1.06	0–3
Model at 6 years	0.92	0.74	0.53	0.32	0–3
Finding at 6 years	0.97	1.09	0.83	−0.60	0–3
Mathematical tasks					
Cardinality skills at 3.5 years	3.00	1.12	1.88	4.16	2–7
Number sequence production					
3.5 years	9.51	5.35	0.40	−0.87	2–19
5 years	23.10	10.98	0.89	0.80	2–50
6 years	36.38	14.85	−0.44	−1.48	8–50
Counting of objects					
5 years	10.67	6.02	0.50	0.25	0–23
6 years	14.18	5.42	0.26	−1.26	5–23
Mathematical sum variable					
3.5 years	0.00	0.92	0.90	0.32	
5 years	0.00	0.91	0.66	0.60	
6 years	0.00	0.87	−0.18	−1.13	

recommended the following approximate cut-off values for rejecting the model: for SRMR greater than 0.09, and for CFI smaller than 0.95. One-sided statistical testing for hypotheses was used throughout the analysis because the direction of all relationships was hypothesized in advance. The path analyses were also confirmed by using normal score-transformed variables.

We began by testing a longitudinal path model based on the hypothesis that the reciprocal relationship between SFON and mathematical skills would predict mathematical skills: better earlier mathematical skills would be associated with

Table 3
Intercorrelations of SFON tendency and mathematical skills in Study 1

Variable	1	2	3	4	5	6
1. SFON at 4	–					
2. SFON at 5	0.53**	–				
3. SFON at 6	0.24	0.15	–			
4. Mathematical skills at 3.5	0.42**	0.35*	0.57**	–		
5. Mathematical skills at 5	0.49**	0.36*	0.51**	0.64**	–	
6. Mathematical skills at 6	0.34*	0.17	0.54**	0.45**	0.60**	–

Note. Two-tailed hypotheses, $*p < 0.05$, $**p < 0.01$.

strong SFON tendency, which in turn would be related to subsequent strong development in mathematical skills. We also expected stability in the development of mathematical skills as well as in SFON tendency during the follow-up.

We started with a model in which mathematical skills were hypothesized to be predicted by immediately previous mathematical skills and by immediately previous or concurrent SFON tendency. All theoretically relevant parameters were estimated and then the process was continued by removing the non-significant paths step by step. The resulting final model, including only statistically significant paths, is presented in Fig. 2 with standardized parameter estimates. The model fitted the data well, $\chi^2(8) = 8.08$, $p = 0.43$, SRMR = 0.068, CFI = 1.00.

As expected, children's mathematical skills at 5 were predicted by their earlier mathematical skills as well as their SFON at 4: the further the children were able to count at 3.5, and the stronger their earlier tendency to focus on numerosity, the better were their mathematical skills at 5. The SFON at 5 was associated only with earlier SFON. The children's mathematical skills at 6 were predicted by their mathematical skills at 5, and associated with their concurrent SFON. The expected path from SFON at 5 to mathematical skills at 6 was not significant; nor was the path from SFON at 5 to SFON at 6, when the effect of mathematical skills at 5 was controlled.

On the contrary, the indirect effects of SFON tendency at 4 via mathematical skills at 5 to mathematical skills at 6 ($0.27 \times 0.60 = 0.16$), as well as to SFON tendency at 6 ($0.27 \times 0.51 = 0.14$) were statistically significant ($p < 0.05$). Moreover, the earlier indirect effect of mathematical skills at 3.5 via SFON at 4 to mathematical skills at 5 ($0.45 \times 0.27 = 0.12$) was significant.

These indirect effects give some support to our hypotheses on the reciprocal nature of the relationship between SFON and mathematical skills. Thus, SFON at 4

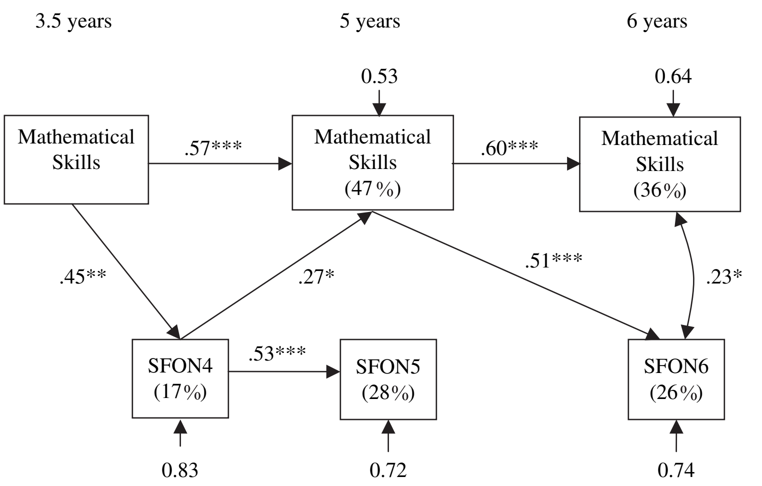

Fig. 2. Path model 1 of observed variables in Study 1.

partly mediated the effect of earlier mathematical skills to mathematical skills at 5, and the mathematical skills at 5 partly mediated the effect of SFON at 4 to both the observed variables at 6. The SFON measured at 6 was significantly associated with concurrent mathematical skills at the same point, and it was also predicted by mathematical skills at 5.

As a second model (see Fig. 3) we tested whether there would be over-time lagged effects of mathematical skills at 3.5 on later measures, and thus also included the possible paths from mathematical skills at 3.5 to the variables measured at 5 and 6. This model fitted the data, $\chi^2(7) = 2.64$, $p = 0.92$, SRMR = 0.041, CFI = 1.00. While dropping non-significant paths, we decided to keep the statistically symptomatical (one-tailed, $p = 0.06$) path coefficient (0.26) from mathematical skills at 5 to SFON tendency at 6 for a better overall fit of the model. The children's SFON tendency at 6 was significantly predicted by their mathematical skills at 3.5 and at 5 (standardized parameter estimates 0.42 and 0.26, respectively). The lagged effect of mathematical skills at 3.5 was not statistically significant for SFON at 5 nor for mathematical skills at 6. The other paths remained significant, and to practically the same extent as in our first model. This model accounted for the same amount of variance as our first model in all other variables except for an increase in the explained variance in SFON tendency at 6 to 35%.

2.3. Conclusions

Children varied in their SFON, and the average measure of intraclass correlations showed stability in their SFON tendency across the SFON measures of the three measurement points. The results showed individual differences in children's SFON related to their mathematical skills. Path models indicated a reciprocal development

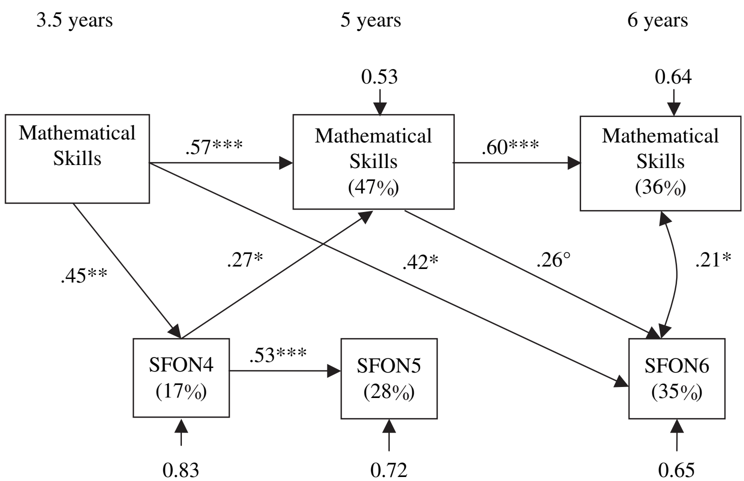

Fig. 3. Path model 2 of the observed variables in Study 1.

between SFON and mathematical skills. Not only was SFON predicted by the mathematical skills, but also mathematical skills were predicted by SFON. Early mathematical skills at 3.5 were related to the SFON at 4, which in turn was either directly or indirectly related to later mathematical skills at 5 and 6, as well as to SFON at 5. The concurrent association between SFON and mathematical skills at 6 was significant in all the models. Interestingly, the lagged direct or indirect effect of mathematical skills measured at 3.5 was significant on the variables measured at 6. The mathematical development showed stability, unlike SFON in the period from 5 to 6 years.

3. Study 2

Our aim was to study SFON within a larger sample research project, in which SFON tasks were presented among non-mathematical tasks. This enabled us to hide the quantitative purpose of SFON tasks. Furthermore, we aimed to study whether SFON would be related to object counting skills, number sequence elaboration, and basic arithmetic skills. In study 2, we specifically tested whether those children who did not spontaneously focus on numerosity, could execute the SFON tasks, when explicitly guided to look at, and to count relevant numbers in the tasks. The aim was to confirm that it is the lack of spontaneous focusing on numerosity, not the lack of adequate counting and other procedural skills, which lead to failure on SFON tasks. Moreover, we wanted to control the effect of non-verbal intelligence or comprehension of instructions on the association between SFON and mathematical skills.

3.1. Method

3.1.1. Participants

The participants were 183 pre-schoolers (102 boys, and 81 girls) aged from 6 years 0 months to 7 years 0 months ($M = 6.5$ years, SD = 4 months). The sample included 18 children from immigrant-families with at least a moderate command of the Finnish language, and 17 children with special educational needs.

Study 2 is part of a larger longitudinal study, and it was carried out in the same town as Study 1. The sample represented an urban Finnish population in day care except for a slight over-representation of boys in the sample.

3.1.2. Procedures

Each child's SFON and mathematical skills were assessed individually at the day-care centre by one of four experimenters in the first testing session, and by one of two experimenters in the second testing session. The SFON assessment was based on structured observations. Before the assessment, the experimenters were fully trained in testing and strategy observation procedures with a videotaped test session and subsequent checking sessions during the testing period. Procedural questions and disagreements were noted and resolved through discussion.

Compared to the SFON tasks presented at the age of 6 in Study 1, two identical and one context-changed SFON tasks were used: the Imitation, Model, and Finding tasks. The testing sessions included three SFON tasks, a variety of non-quantitative tasks, and three mathematical tasks. Two SFON tasks were presented at the beginning of the testing session, and one after two non-quantitative tasks. The mathematical tasks ended the testing sessions, except for a sub-sample who were confronted with Guided focusing versions of the SFON tasks. Comprehension of verbal instructions was measured by a standardized neuropsychological test for children on a separate session (NEPSY, by Korkman, Kirk, & Kemp, 1997). The Raven's non-verbal intelligence test was administered a year before the other data (Raven, 1976).

During the SFON tasks, the experimenter filled in an observation form based on criteria for SFON in Study 1 on each child's trial. There were no differences in numbers, or quality of observations of individual experimenters ($p > 0.05$). The average intraclass correlation for the three SFON tasks was 0.46, corresponding to the results of Study 1.

3.1.3. SFON tasks

Children were confronted with the Imitation, Model and Finding tasks of Study 1.

The materials of the Imitation task were a blue parrot, capable of swallowing, placed in front of the child on the table, and a box of 10 red glass sweets placed on the left, and a box of 10 blue glass sweets, in front of the parrot. The procedure for the task was similar to the Post-box task in Study 1. Only the parrot replaced the post-box, and the sweets the envelopes.

The Model and Finding tasks were identical to the tasks in Study 1.

3.1.4. Mathematical tasks

3.1.4.1. Counting of disarranged objects. The measure of object counting was identical to the object counting measure of Study 1 except that the numbers of objects were 5, 9, 13, 19, and 23.

3.1.4.2. Number sequence elaboration. We used a modified version of the Salonen et al. (1994) test to assess children's number sequence elaboration skills. The child was asked to (1) count forwards from 3, 8, 12, 19, and 24, (2) count forwards from 2 to 7, from 6 to 11, from 14 to 19, and from 18 to 25, (3) count backwards from 4, 8, 12, 19, 24, and (4) count backwards from 6 to 3, from 13 to 8, from 19 to 15, and from 23 to 18. The child's counting was stopped after four number words in tasks requiring continual counting. The child was allowed to spontaneously correct his or her attempt once. The maximum score on the task, 18 points, was achieved by producing the correct number words for the items.

3.1.4.3. Basic arithmetic skills. There were two tasks each for addition and subtraction operations. To minimise linguistic comprehension difficulties the tasks were presented using glass (1.5 cm in diameter) sweets and a non-transparent box. In the addition task, the interviewer showed 4 glass sweets in her hand to the child saying: "There are 4 sweets over here. I put them under this box. Then I put 3 more sweets there. How many sweets are there under the box now?" The second item for the addition operation was 4 added to 6. In the following subtraction task, the child was asked to look at 6 sweets on the table. The sweets were covered by a box, and then 3 sweets were taken away from the box. The child was not able to see the remaining sweets. The child was asked how many sweets there were in the box now. The second subtraction item was 4 taken away from 9. Giving accurate answers to the tasks produced altogether four points.

3.1.5. Controlling measures

3.1.5.1. Raven's non-verbal intelligence test. Non-verbal intelligence was measured individually by the Raven's Coloured Matrices (Raven, 1976).

3.1.5.2. Comprehension of verbal instructions. Comprehension of complex verbal instructions was assessed individually by the Token test of standardized neuropsychological test for children (NEPSY, by Korkman et al., 1997). This is a receptive language test, which also gauges attention in children.

3.1.5.3. Guided focusing versions of the SFON tasks. After all the mathematical tasks, a sample of those children who did not focus their attention on numbers of objects at all, either in the Imitation, Model, or Finding task, were given a Guided Focusing (GFON) version of a corresponding SFON task by one experimenter. The sample was randomly selected from all the participating day-care centres. In addition, it included all the children with special educational needs. The numbers of children (out of all the children who did not focus on numerosity in the corresponding SFON task) who were given a GFON version of the Imitation, Model, or Finding task were 11 out of 26 (42%), 15 out of 48 (31%) and 27 out of 70 (39%), respectively.

At the beginning of the GFON version of the corresponding SFON task, the child was told that he or she would do the old SFON task (e.g. "The Dinosaur task") in a new way. This time the child should watch, and count, how many sweets (or stamps) the experimenter would use, and then produce exactly as many sweets (or stamps) by him- or herself. In the GFON version of the Finding task, the child was advised to look at, and count, which, in order, was the hat covering the ingot. No hints for memorising the numbers were given. The GFON tasks included the same three items as the corresponding SFON tasks. Scoring was based on accurate numerosity produced.

3.2. Results

3.2.1. SFON

Table 4 presents the numbers of children focusing on numerosity in individual SFON tasks. The results correspond to the results of Study 1 ($p > 0.05$).

Table 4
Frequencies of spontaneous focusing on numerosity in the SFON tasks of Study 2 ($N = 183$)

Number of SFON scores	Imitation	Model	Finding[a]
0	26 (14%)	48 (26%)	70 (38%)
1	26 (14%)	75 (41%)	47 (26%)
2	55 (30%)	47 (25%)	40 (22%)
3	76 (41%)	13 (7%)	24 (13%)

Note. $N = 183$.
[a] 181 children participated in the Finding task.

A sum variable describing the children's spontaneous overall tendency to focus on numerosity across the three SFON tasks was used for further analyses (see Table 5). The maximum of the SFON sum variable was 9. The children focused on numerosity on average 4.23 times (SD = 2.10) altogether in the SFON tasks. The only gender difference in the Study 2 was found in the Imitation task, where girls focused more on numerosity than boys ($p < 0.01$).

3.2.2. Relationship between mathematical skills and SFON

Descriptive statistics of the variables are presented in Table 5. Statistically significant Pearson's correlation coefficients (see Table 6) show that children's SFON tendency was positively associated with their mathematical skills, with number sequence elaboration, $r = 0.42$, with counting of objects, $r = 0.35$, and with basic arithmetic skills, $r = 0.49$ (values of $p < 0.01$). Whether this association was explained by non-verbal IQ or verbal comprehension was studied by partial correlations. Controlling non-verbal IQ, or comprehension of verbal instructions affected the association between SFON and mathematical skills only slightly. When the effects of non-verbal IQ and comprehension of verbal instructions were controlled, the partial correlations of SFON to object counting was pr = 0.26, to number sequence elaboration pr = 0.27, and to basic arithmetic skills pr = 0.37 (values of $p < 0.001$).

Table 5
Descriptives of the variables in Study 2

Variable	*M*	SD	Skewness	Kurtosis	Range
SFON sum score	4.23	2.10	−0.56	−0.44	0–9
Imitation	1.99	1.06	−0.70	−0.78	0–3
Model	1.10	1.07	0.47	−1.08	0–3
Finding	1.14	0.89	0.35	−0.65	0–3
Mathematical skills					
Counting of objects	14.73	6.53	0.76	−1.513	5–23
Number sequence elaboration	13.95	4.24	−1.47	1.69	0–18
Basic arithmetic skills	2.68	1.27	−0.58	−0.78	0–4
Controlling measures					
Non-verbal IQ (raw scores)	18.90	4.84	0.32	−0.95	9–33
Comprehension of instructions	18.54	4.18	−0.528	0.593	6–27

Table 6
Correlations of the variables in Study 2 ($n = 181$)

Variable	1	2	3	4	5	6
1. SFON tendency	–					
2. Number sequence elaboration	0.42**	–				
3. Counting of objects	0.35**	0.34**	–			
4. Basic arithmetic skills	0.49**	0.64**	0.37**	–		
5. Non-verbal IQ by Raven	0.46**	0.41**	0.26**	0.38**	–	
6. Comprehension of instructions[a]	0.25**	0.48**	0.14	0.39**	0.38**	–

Note. **$p < 0.01$, two-tailed hypotheses.
[a] $n = 177$.

3.2.3. Guided focusing on numerosity

To study whether those children who did not focus their attention on numbers in the SFON task, would be able to execute the SFON tasks when advised to focus on numerosity, their results on GFON tasks were compared with their results on SFON tasks. The mean of the total scores for the GFON version of the Imitation task was 1.55 (SD = 1.13). There were two children who were not able to carry out the GFON version of the Imitation task. These children could not memorise the numbers of sweets accurately. However, they were successful in an extra attempt, where the experimenter also asked them to print in their minds the number of sweets they should give the bird. In the GFON version of the Model task the sample mean was 2.60 (SD = 0.63). All children were able to perform at least one item of the task. In the GFON version of the Finding task, the children's mean for total scores was 2.07 (SD = 0.96). Two children violated the one to one correspondence in counting the hats, and were not able to locate the target hat accurately. All the four unsuccessful children in the GFON tasks (altogether 44 children were presented with one or more GFON task) had been earlier diagnosed as children with special educational needs, because of either linguistic or attentional deficiency. If these four children's failed performances are excluded, the overall success percentage for children on items of individual GFON tasks was 63% for the Imitation task, 85% for the Model task and 75% for the Finding task. This indicates that differences in children's performance on the SFON tasks were not the result of procedural difficulties, although in the groups of most disadvantaged children there were some such difficulties.

3.3. Conclusions

In accordance with the results in Study 1, children in Study 2 had remarkable differences in their spontaneous focusing on numerosity. The children's SFON tendency was related to their object counting, number sequence elaboration and basic arithmetic skills. This confirms that spontaneous focusing on numerosity is involved in the mathematical development even after children have achieved basic cardinality skills. The partial correlations showed that the shared variance in SFON and mathematical skills was domain-specific, and not explained by children's differences in non-verbal intelligence and comprehension of verbal instructions.

The results of the GFON tasks showed that children particularly lacked focusing on the numerical aspect of the SFON task, not the quantifying means or other cognitive skills needed in the task. However, the results of the GFON tasks indicated the existence of a few children with special educational needs, who might have difficulties in counting and retaining numbers in the SFON tasks.

4. General discussion

The results of the studies give support to our main hypotheses concerning the existence and stability of SFON, as well as its positive association with mathematical skills. Hence, we conclude that within a child's existing mathematical competence, it is possible to distinguish a separate process, which refers to the child's tendency to spontaneously focus on numerosity. We also found considerable stability in this tendency across different tasks. Furthermore, SFON tendency seems to be clearly positively related to the development of early mathematical skills. As these results are based on both longitudinal and cross-sectional multi-method data of different samples, and the results showed similar patterns, there is reasonable support for the validity of the main findings of the study.

The three main findings were supported by both the studies. Furthermore, the results of Study 1 indicated stability in SFON across time at the age of 4–6 years, as well as the reciprocal nature of the relationship between SFON and mathematical skills. The results of Study 2 extended these findings by showing, firstly, that the covariance of mathematical skills and SFON tendency was not explained by non-verbal IQ or comprehension of verbal instructions. Secondly, Study 2 showed that the children's failure in SFON tasks was not caused by their inability to deal with the cognitive requirements of these tasks.

In order to receive a reliable indicator for a child's general SFON tendency in everyday situations, several SFON measures within different contexts are needed. We acknowledge that our tasks measured SFON in a limited number of situations. Structured observations of children's SFON in everyday surroundings in comparison with their performance on SFON tasks would show whether these two are actually related. Observational studies on young children's mathematical activities by Ginsburg, Balfanz, and Greenes (2000) have indicated that mathematical-seeming behaviour appears in children's free play, although individual differences, or intentions of the playing children have remained unclear. The structured SFON tasks may reveal young children's focusing on numerosity better than observations of free activity, since the probability of producing exactly accurate numbers in SFON tasks by accident is very low. The observational data on the specific signs of the child's considering numerosity or quantitative goals reveal SFON also when the child makes mistakes in enumeration.

The results of our longitudinal Study 1 are partly tentative because of the small sample size and the fact that the mathematical purpose of the testing was not totally hidden from the children. They possibly remembered something from the previous testing procedures although no feedback about the mathematical nature of the SFON tasks was ever given to them. However, in Study 2, tasks from various non-quantitative domains presented together with SFON tasks covered the mathematical nature of the SFON tasks well, and the results of the corresponding SFON tasks in Study 1 were replicated. This allows us to conclude that the longitudinal project did not significantly compromise the reliability of SFON measures.

The average measure intraclass correlations showed reasonable overall stability in SFON across the three SFON measurement points in Study 1. The stability hypotheses were also partly supported by the path models, which showed stability in SFON from the age of 4–5 years. It is possible that the instability shown by the path models, between SFON at the age of 6 and the previous SFON measures indicate differences in the SFON measures, or possibly different utilizing processes for enumeration in subitizable and countable set sizes. These questions remain unanswered, but they should be studied in the future.

Theoretical implications of the study indicate we should regard SFON as a separate, but closely related factor to enumeration skills. In concert with Sophian's (1998) views, during the childhood years, conceptual and procedural advances in enumeration enable and facilitate the child's SFON, which in turn leads to the child's own practice in utilizing numerosity in a variety of situations, and subsequently develops enumeration skills further. The results of Study 1 showed that SFON and mathematical skills are developmentally intertwined, but no conclusions on the developmental priority can be made without further studies with an experimental design.

The findings of the study raise interesting new questions for future research. A weak SFON tendency seems to be related to delayed mathematical development at an early age. Would it be possible to prevent mathematical difficulties by enhancing children's focusing on numerosity? Or are differences in children's focusing on numerosity present already in infants? It would be of a particular interest to broaden our knowledge about the role of cultural environment, as well as that of adults (see Saxe et al., 1987) in how children learn to focus on numerosity and formulate the goals of quantitative tasks in social interaction. Moreover, studying children's subitizing-based enumeration in relation to SFON and counting development would improve our understanding of the integration of biologically and culturally based enumeration skills.

This study adds to our current limited knowledge of children's spontaneous practice in utilizing enumeration. Our findings emphasise the importance of regarding SFON also in the assessments of young children's mathematical skills. Mathematical assessments can be misleading if the child's attention has not been clearly enough guided towards numerical aspects. It is of great educational relevance whether the child lacks the enumeration skills required or just does not focus on numerosity in the task.

When designing learning environments for practising mathematical skills it is important to consider how mathematically children interpret the tasks, and especially to confirm that the mathematically less skilled children actually focus on the numerical aspects. Future experimental studies can explore whether strengthened SFON tendency and advances in numerical skills together form a positive cumulative cycle.

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